Statistics

Lecture 6

Feb 19-8:47 AM

More on Probabilities:

A piggy bank has 3 nickels and 2 dime.

Take 2 coins with replacement.

N → Nickel

D → Dime

NN
ND
DN
DD

Sample Space → Complete list of all possible outcomes.

NN → 10¢ $P(10¢) = P(NN) = \frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25} = \boxed{.36}$

ND, DN → 15¢ $P(15¢) = P(ND \text{ or } DN) = 2 \cdot \frac{3}{5} \cdot \frac{2}{5} = \frac{12}{25} = \boxed{.48}$

DD → 20¢ $P(20¢) = P(DD) = \frac{2}{5} \cdot \frac{2}{5} = \frac{4}{25} = \boxed{.16}$

Total ¢	P(Total ¢)
10¢	.36
15¢	.48
20¢	.16

clear all lists.

Total ¢ → L1 , P(Total ¢) → L2

use $\boxed{\text{1 - Var Stats}}$ with L1 & L2

$\bar{x} = 14$

$S = S_x =$ Blank

Total Prob. → $n = 1$

Apr 3-8:03 AM

There are 3 females and 5 males.

we need to $\underbrace{\text{form of group of 2 people.}}$

Select 2   No replacement

F → Female
M → Male

FF
FM
MF
MM

Sample Space

FF → 2 females $P(2\ \text{Females}) = \frac{3}{8} \cdot \frac{2}{7} = \boxed{\frac{3}{28}}$

FM
MF → 1 Female $P(1\ \text{Female}) = 2 \cdot \frac{3}{8} \cdot \frac{5}{7} = \boxed{\frac{15}{28}}$

MM → 0 Females $P(0\ \text{Females}) = \frac{5}{8} \cdot \frac{4}{7} = \frac{5}{14} = \boxed{\frac{10}{28}}$

# Females	P(# Females)
2	$\frac{3}{28}$
1	$\frac{15}{28}$
0	$\frac{10}{28}$

Clear all lists

Females → L1, P(# Females) → L2

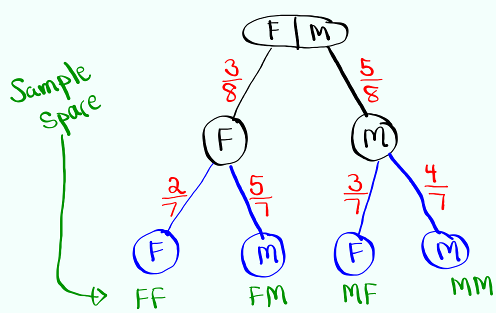
use $\boxed{\text{1-Var Stats}}$ with

L1 & L2

$\bar{x} = .75$ $S = S_x$ blank $n = 1$

Total Prob.

Apr 3-8:15 AM



Let's Select 3 people

FFF

Some F & Some M

MMM

$P(\text{all Females}) = \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{1}{6} = \frac{1}{56}$

$P(\text{all Males}) = \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} = \frac{5}{28}$

$P(\text{at least 1 Female}) = 1 - P(\text{No Females})$

$= 1 - P(\text{all males})$

$= 1 - \frac{5}{28} = \boxed{\frac{23}{28}}$

$P(\text{at least 1 male}) =$

$1 - P(\text{No males}) =$

$1 - P(\text{all Females}) = 1 - \frac{1}{56} = \boxed{\frac{55}{56}}$

Apr 3-8:27 AM

Given $P(A) = .4$ $\quad P(B) = .5$

Find $P(A \text{ and } B)$

1) If $\boxed{A \text{ \& } B \text{ are mutually exclusive events}}$.

$P(A \text{ and } B) = \boxed{0}$

2) If $\boxed{A \text{ \& } B \text{ are independent events}}$

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

$$= (.4)(.5) = \boxed{.2}$$

Apr 3-8:38 AM

Given $P(A) = .3$, $P(B) = .5$, A & B are independent events

1) $P(\bar{A}) = 1 - .3 = \boxed{.7}$

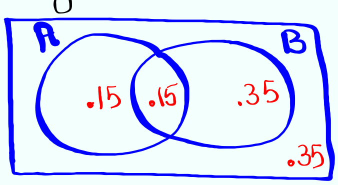
2) $P(A \text{ and } B) = P(A) \cdot P(B) = \boxed{.15}$

3) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$.3 + .5 - .15 = \boxed{.65}$

4) Construct Venn Diagram.

Total = 1



5) $P(\bar{A} \text{ and } \bar{B}) = P(\overline{A \text{ or } B}) = \boxed{.35}$

De Morgan's Law

6) $P(\bar{A} \text{ or } \bar{B}) = P(\overline{A \text{ and } B}) = \boxed{.85}$

Apr 3-8:42 AM

General Multiplication Rule

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

A happens

then B happens

Given

If we isolate $P(B \mid A)$

$$\boxed{P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}}$$

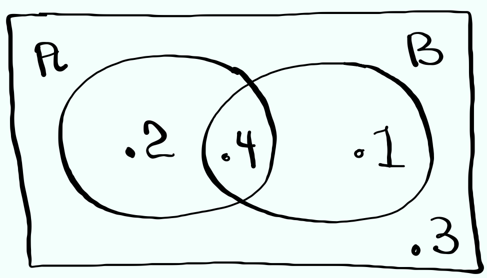
Conditional Prob.

Apr 3-8:50 AM

$P(A) = .6$

$P(B) = .5$

$P(A \text{ and } B) = .4$



$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{.4}{.6} = \frac{2}{3} = \boxed{.667}$$

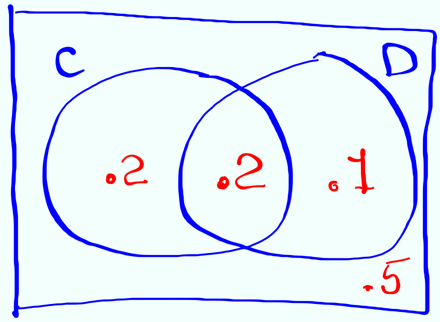
$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{.4}{.5} = \frac{4}{5} = \boxed{.8}$$

Apr 3-8:53 AM

$P(\text{Coffee}) = .4$

$P(\text{Donut}) = .3$

$P(\text{Coffee and Donut}) = .2$



$$P(\text{Donut} \mid \text{Coffee}) = \frac{P(C \text{ and } D)}{P(C)} = \frac{.2}{.4} = \frac{1}{2} = \boxed{.5}$$

$$P(\text{Coffee} \mid \text{Donut}) = \frac{P(C \text{ and } D)}{P(D)} = \frac{.2}{.3} = \frac{2}{3} = \boxed{.667}$$

Apr 3-8:58 AM

$P(A) = .6$

$P(B) = .5$

$P(A \mid B) = .8$

$P(A \text{ and } B) = P(A) \cdot P(B)$ if they are indep. events

we know they are Dep. events

1) $P(A \text{ and } B)$

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

$$.8 = \frac{P(A \text{ and } B)}{.5}$$

Cross-multiply

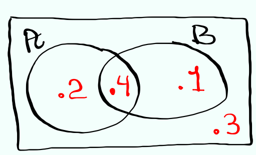
$P(A \text{ and } B) = (.8)(.5) = \boxed{.4}$

2) $P(B \mid A) = \dfrac{P(A \text{ and } B)}{P(A)} = \dfrac{.4}{.6} = \dfrac{2}{3} = \boxed{.667}$

3) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$= .6 + .5 - .4 = \boxed{.7}$

4) Construct Venn Diagram.



Apr 3-9:04 AM

4 Females, 6 Males

10 positions

5 Morning, 3 afternoon, 2 grave yard

P(at least 1 Female in the afternoon shift)

$= 1 - P(\text{No Females in the afternoon shift})$

$= 1 - P(\text{all males in the " "})$

$= 1 - \frac{6}{10} \cdot \frac{5}{9} \cdot \frac{4}{8} = \boxed{\frac{5}{6}}$

P(at least 1 male in the grave yard shift)

$= 1 - P(\text{No males}) = 1 - P(\text{all Females}) = 1 - \frac{4}{10} \cdot \frac{3}{9}$

$= \boxed{\frac{13}{15}}$

SG 13 ✓

Apr 3-9:13 AM

Data

1) Qualitative

2) Quantitative
- 1) Discrete
- 2) Continuous

Apr 3-9:41 AM

Let x be a discrete random variable with prob. dist. $P(x)$.

Prob. dist. provides the prob. of all events in the Sample Space.

1) in the form of a chart
2) using a formula
3) using a graph
4) we could use def. of prob.

Some rules

1) $0 \le P(x) \le 1$
2) $\sum P(x) = 1$
3) $P(x) = 0 \leftrightarrow$ Impossible event
4) $P(x) = 1 \leftrightarrow$ Sure event
5) $0 < P(x) \le .05 \leftrightarrow$ Rare event

Apr 3-9:43 AM

Consider the chart below

x	$P(x)$
1	.2
2	.5
3	.3

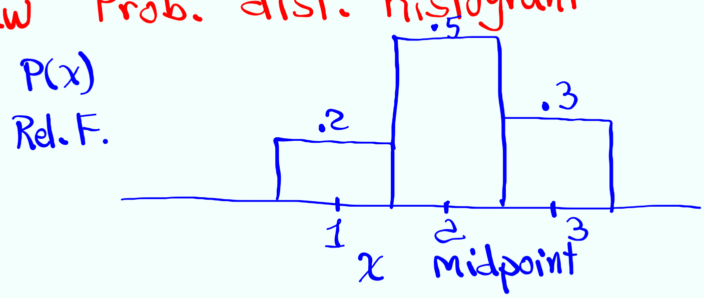
1) verify $\sum P(x) = 1$ ✓

$.2 + .5 + .3 = 1$ ✓

2) $P(x \le 2) = .2 + .5 = .7$

3) $P(x \ge 2) = .5 + .3 = .8$

4) Draw Prob. dist. histogram



Apr 3-9:49 AM

Consider the chart below:

x	$P(x)$
1	.2
2	.3
3	.4
4	.1

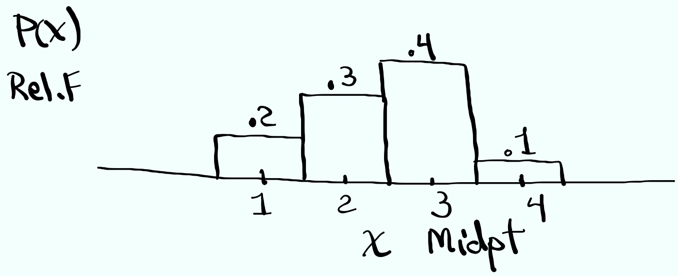
1) $P(x=4) =$

$1 - [.2 + .3 + .4] = \boxed{.1}$

2) $P(x=2 \text{ or } x=4)$

$= .3 + .1 = \boxed{.4}$

3) Draw Prob. dist. histogram



Apr 3-9:53 AM

Complete the chart below

x	$P(x)$	$x \cdot P(x)$	$x^2 \cdot P(x)$
1	.2	.2	.2
2	.5	1.0	2.0
3	.3	.9	2.7

1) $\sum P(x) = 1$

2) $\sum x p(x) = 2.1$

3) $\sum x^2 p(x) = 4.9$

4) Compute $\sum x^2 p(x) - \left(\sum x p(x)\right)^2$

$= 4.9 - 2.1^2 = \boxed{.49}$

5) $\sqrt{\text{Last answer}} = \sqrt{.49} = \boxed{.7}$

Apr 3-9:58 AM

3 nickels & 2 Quarters

Take 2 Coins with replacement

NN $\rightarrow$ 10¢ $\quad P(10¢) = \frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25} = .36$

NQ
QN $\rightarrow$ 30¢ $\quad P(30¢) = 2 \cdot \frac{3}{5} \cdot \frac{2}{5} = \frac{12}{25} = .48$

QQ $\rightarrow$ 50¢ $\quad P(50¢) = \frac{2}{5} \cdot \frac{2}{5} = \frac{4}{25} = .16$

x	$P(x)$	$xP(x)$	$x^2P(x)$
10	.36	3.6	36
30	.48	14.4	432
50	.16	8.0	400

$\sum P(x) = 1 \qquad \sum xp(x) = 26 \qquad \sum x^2 p(x) = 868$

Compute $\sum x^2 p(x) - \left(\sum x p(x)\right)^2 = 868 - 26^2 = 192$

$\sqrt{\text{Last answer}} = \sqrt{192} \approx \boxed{14}$

Apr 3-10:03 AM

Mean $\quad \mu \quad$ mu $\quad \mu = \sum x p(x)$

Variance $\quad \sigma^2 \quad$ Sigma^2 $\quad \sigma^2 = \sum x^2 p(x) - \mu^2$

Standard deviation $\quad \sigma \quad$ Sigma $\quad \sigma = \sqrt{\sigma^2}$

Using TI

$x \rightarrow L1$

$P(x) \rightarrow L2$

Use $\boxed{\text{1-Var Stats}}$ with L1 & L2

$\mu = \bar{x}$

$\sigma = \sigma_x$

$n = 1$

Apr 3-10:15 AM

x	$P(x)$
1	.2
2	.5
3	.3

$x \to L1$

$P(x) \to L2$

$\boxed{\text{1-Var Stats}}$ with L1 & L2

$\mu = \bar{x} = 2.1$

$\sigma = \sigma_x = .7$ $\to \sigma^2 = .49$

$n = 1$ ← Total Prob.

what about σ^2?

$\boxed{\text{VARS}}$ $\boxed{\text{5: Statistics}}$ $\boxed{4: \sigma_x}$ $\boxed{x^2}$ $\boxed{\text{Enter}}$

Apr 3-10:20 AM

x	$P(x)$
1	.3
2	.2
3	.4
4	.1

$x \to L1$

$P(x) \to L2$

$\boxed{\text{1-Var Stats}}$ with L1 & L2

$\mu = \bar{x} = 2.3$

$n = 1$ ✓

$\sigma = \sigma_x = 1.005$

$\frac{101}{100}$ ← σ^2

$\boxed{\text{VARS}}$ $\boxed{\text{5: Statistics}}$ $\boxed{4: \sigma_x}$ $\boxed{x^2}$ $\boxed{\text{Math}}$ $\boxed{\text{1:▶Frac}}$

$\boxed{\text{Enter}}$

Apr 3-10:24 AM

A prob. dist. has a mean of 16 and standard. dev. of 4.

$\mu = 16$

$\sigma = 4$

68% Range $\rightarrow \mu \pm \sigma = 16 \pm 4 \rightarrow \boxed{12 \text{ to } 20}$

Usual Range $\rightarrow \mu \pm 2\sigma = 16 \pm 2(4) \rightarrow \boxed{8 \text{ to } 24}$

Apr 3-10:29 AM

Application

Buy a ticket for \$10

18 Tickets Sold

1 tKT Drawn $\rightarrow$ winner gets a TI-84 Calc. worth \$130.

Net	P(Net)	
10 − 130	1/18	win. tkt
10 − 0	17/18	$\overline{\text{win}}$ TKT

Expected Value / TKT

$\boxed{\mu \approx \$2.78}$

$\bar{x}$

Net $\rightarrow$ L1

P(Net) $\rightarrow$ L2

Apr 3-10:32 AM

You buy insurance for \$50

Any damages → You will get \$500

Prob. of any damages is .02

Net	P(Net)	
50 − 500	.02	Damage
50 − 0	.98	$\overline{\text{Damage}}$

E.V./Policy Sold

Net → L1

P(Net) → L2

$E.V. = \mu = \overline{x}$

$\boxed{\$40}$

Apr 3-10:40 AM